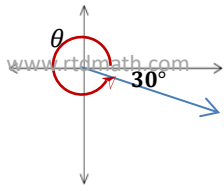


- 1.** First determine quadrant θ terminates in. Since \sin is negative in Quad III and IV, and \tan is neg. in II and IV, θ is in **quad IV**.



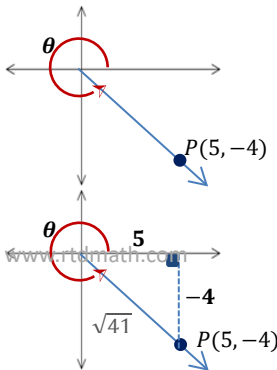
$$\begin{aligned} \text{So, } \theta &= 360^\circ - 30^\circ \\ &= 330^\circ \text{ or } \frac{11\pi}{6} \text{ (not one of the choices)} \end{aligned}$$

So, consider co-terminal angles of 330° by adding / subtracting 360° (and converting to radians), or use $\frac{11\pi}{6}$ and add/subtract 2π ...

$$= 330^\circ + 360^\circ = 690^\circ, \text{ converts to } \frac{23\pi}{6}$$

ANSWER: **D**

- 3.** First determine quadrant θ terminates in. Since \sin is negative in Quad III and IV, and \tan is neg. in II and IV, θ is in **quad IV**.



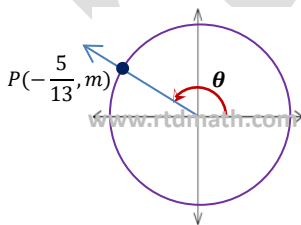
Next, plot a point P in quad IV and sketch angle θ

Then make a triangle by connecting P to the x -axis. Label the sides of the triangle from the coordinates of P ... use pyth. theorem to get the hyp.

$$\text{Now use } \sec\theta = \frac{1}{\cos\theta} \text{ where } \cos\theta = \frac{\text{adj}}{\text{hyp}} \dots \text{so } \sec\theta = \frac{\text{hyp}}{\text{adj}}$$

$$\sec\theta = \frac{\sqrt{41}}{5} \Rightarrow \approx 1.28 \quad \text{ANSWER: } \mathbf{A}$$

- 4.** There are two options for $P(-\frac{5}{13}, m)$, it can be drawn in quadrant II or III. (as the x -coord is negative) However, it is given that \tan is negative, we know we should draw P in **quad II**. At this point we can pause to consider how fun it is to reason things out like that. (pause for 10 to 15 seconds)



← Diagram of information given

We can now solve for m by either drawing a triangle and label the hypotenuse 1 (since $-$ unit circle), or by going straight to the unit circle formula:

$$x^2 + y^2 = 1$$

$$\left(-\frac{5}{13}\right)^2 + m^2 = 1 \Rightarrow m^2 = 1 - \frac{25}{169} \Rightarrow m^2 = \frac{144}{169} \Rightarrow m = \pm \sqrt{\frac{144}{169}}$$

$$\Rightarrow m = \pm \frac{12}{13} \quad \Rightarrow m = \frac{12}{13} \quad \text{ANSWER: } \mathbf{C}$$

Since quad II, use the $+$ version!

- 2.** This is a reasoning question, not a calculation question! Using CAST rule, since \cos is negative in Quad II and III, and \tan is neg. in II and IV, θ is in **quad II**.

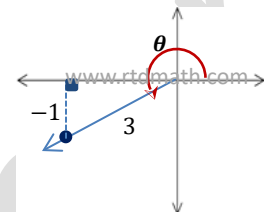
Which means, θ is between $\frac{\pi}{2}$ and π . (1.57 rads and 3.14 as decimals)

So, the only possible option there is **2.62** (Note, all answers are presumed to be in radians as no unit is specified!)

ANSWER: **B**

- NR#1** Since \sin is negative in Quad III and IV, and we want the *smallest* option for θ draw an angle in st. pos. in **quad III**.

Given: $\sin\theta = \frac{-1}{3}$



First find reference angle, let's call it α (inside the triangle)

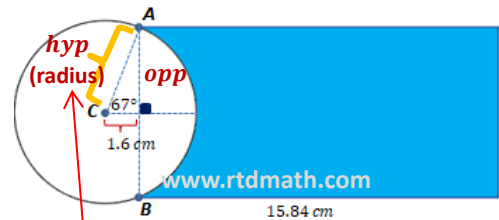
$$\alpha = \sin^{-1}\left(\frac{1}{3}\right)$$

NOTE: When finding reference angles (by definition less than 90° so all trig ratios are positive) **drop any negatives**

NORMAL	FLOAT	AUTO	REAL	RADIAN	MP
$\sin^{-1}(1/3)$ (radian mode)					
0.3398369095					
$\pi + \text{Ans}$					
3.481429563					

$$\theta \approx 3.5 \quad \text{ANSWER: } \mathbf{3.5}$$

- 5.**



First find the length of :

$$\cos 67^\circ = \frac{1.6}{AC} \Rightarrow AC = \frac{1.6}{\cos 67^\circ} \Rightarrow AC \approx 4.095 \text{ (radius)}$$

Next find the length of : (half of AB is the opp side ...)

$$\begin{aligned} \tan 67^\circ &= \frac{\text{opp}}{1.6} \Rightarrow \text{opp} = 1.6 \tan 67^\circ \Rightarrow \text{opp} \approx 3.769 \\ &\Rightarrow AB \approx 2 * 3.769 \Rightarrow AB \approx 7.539 \end{aligned}$$

Finally find the arc length:

$$a = r\theta \Rightarrow a = 4.095 * \frac{(67^\circ * 2)\pi}{180^\circ} \Rightarrow \text{arc} \approx 9.577 \text{ cm}$$

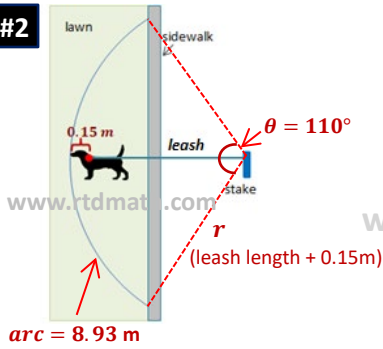
θ in radians

So perimeter is: $2 * 15.84 + 7.54 + 9.58$

Rectangle top / bottom Third rectangle side, AB Final "side", the arc length

$$\text{Perimeter} \approx 48.8 \text{ cm} \quad \text{ANSWER: } \mathbf{C}$$

NR#2



$$\theta = \frac{a}{r}$$

$$r = \frac{a}{\theta}$$

re-arrange

$$r \approx \frac{8.93}{1.92}$$

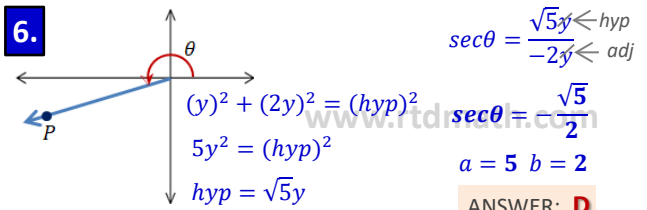
convert θ to radians:
 $\frac{110^\circ \pi}{180^\circ}$

$$r \approx 4.65$$

So, leash+0.15 \approx 4.65
Remember, dog's extra reach is included here!

ANSWER: 4.5

6.



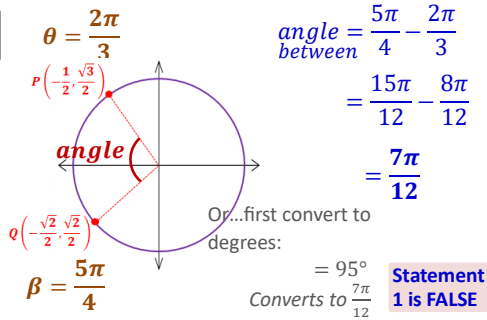
$$\sec \theta = \frac{\sqrt{5}y \leftarrow \text{hyp}}{-2y \leftarrow \text{adj}}$$

$$\sec \theta = \frac{\sqrt{5}}{-2}$$

$a = 5$ $b = 2$

ANSWER: D
Add: $5 + 2 = 7$

7.



$$\text{angle between} = \frac{5\pi}{4} - \frac{2\pi}{3}$$

$$= \frac{15\pi}{12} - \frac{8\pi}{12}$$

$$= \frac{7\pi}{12}$$

$$\theta = \frac{2\pi}{3}$$

Principal angle

$$\theta = \frac{2\pi}{3} - 2\pi$$

is co-terminal

$$\theta = -\frac{4\pi}{3}$$

Statement 2 is TRUE

y-coord at $\frac{2\pi}{3}$ is $\sin \theta$
x-coord at $\frac{5\pi}{4}$ is $\cos \theta$

$$\tan \frac{2\pi}{3} = \frac{\frac{\sqrt{3}}{2} \leftarrow y}{-\frac{1}{2} \leftarrow x}$$

$$\cot \frac{5\pi}{4} = \frac{-\frac{\sqrt{2}}{2} \leftarrow x}{-\frac{\sqrt{2}}{2} \leftarrow y}$$

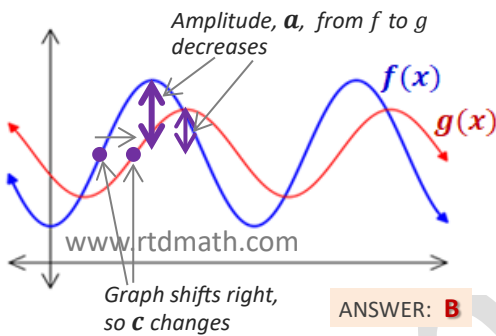
$$= \frac{\sqrt{3}}{2} * \frac{-2}{1} = -\sqrt{3}$$

$$= \frac{-\sqrt{2}}{2} * \frac{-2}{-\sqrt{2}} = 1$$

Statement 3 is TRUE
Statement 4 is TRUE

ANSWER: C

8.



ANSWER: B

9.

First factor b for horiz. phase shift:

$$y = 5 \sin[4(x + \frac{\pi}{4})]$$

Period is $\frac{2\pi}{b}$
Phase shift
 $= \frac{2\pi}{4} \rightarrow = \frac{\pi}{2}$ (to the left, but not relevant)

ANSWER: A

NR#4

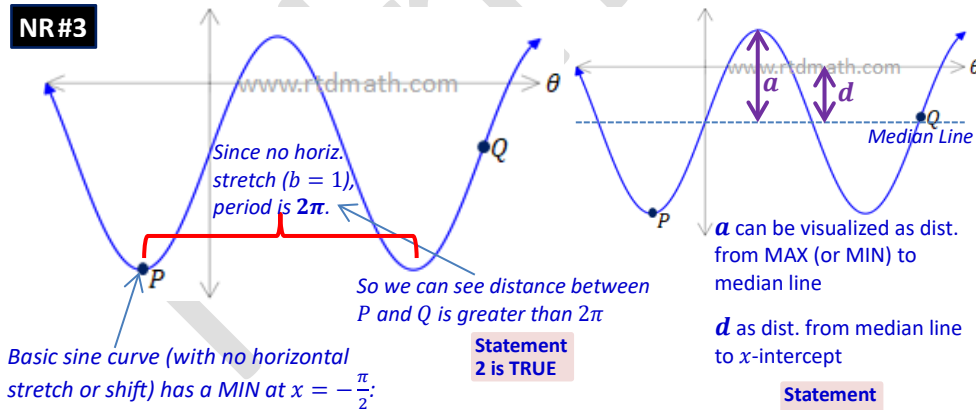
$$y = 2.3 \sin(0.1208x - 0.3624) + 5.8$$

MAX is
AMPL + Vert. Shift
 $= a + d$
 $= 2.3 + 5.8$
 $= 8.1$

Period is $\frac{2\pi}{b}$
 $= \frac{2\pi}{0.1208} \rightarrow = 52$

ANSWER: 5281

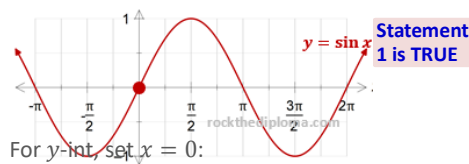
NR#3



Statement 2 is TRUE

Statement 3 is FALSE

Basic sine curve (with no horizontal stretch or shift) has a MIN at $x = -\frac{\pi}{2}$.



For y -int, set $x = 0$:

But $\sin(0) = 0$, so...

$$y = a * \sin(0) - d$$

$$y = -d$$

Statement 4 is FALSE

For $b < 1$, for example...

$$y = a \sin(2x) - d$$

The horiz. str. would be the reciprocal, or here 0.5

ANSWER: 125

Note: Mistake on some answer keys

For horiz str < 1 , all points move closer to the x -axis ...

Statement 5 is TRUE

10.

Domain of $y = \tan x$ is $x \neq \frac{\pi}{2} + n\pi$
That is, where $\cos x = 0$, since $\tan x = \frac{\sin x}{\cos x}$
... at the top / bottom of the unit circle

So for $y = \tan 4x$
Hor. Str. of $\frac{1}{4}$

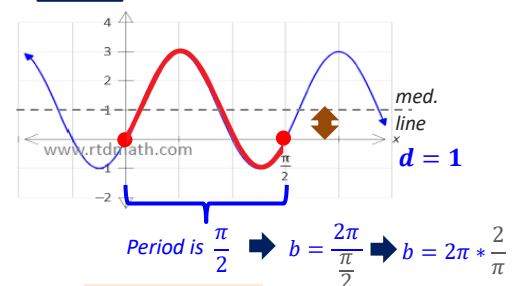
Domain is:

$$x \neq \frac{1}{4} * \frac{\pi}{2} + \frac{1}{4} * n\pi$$

$$x \neq \frac{\pi}{8} + \frac{n\pi}{4}$$

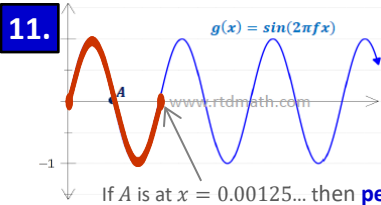
ANSWER: D

NR#5



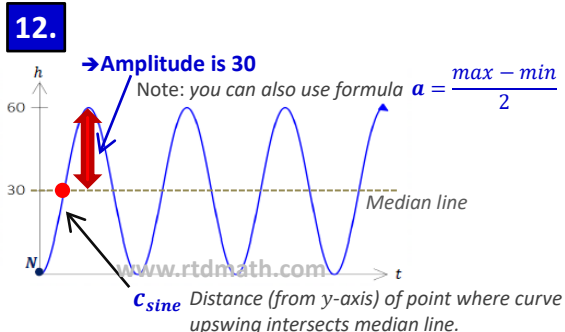
ANSWER: 41

$b = 4$



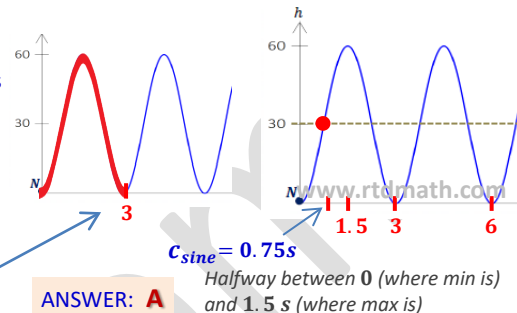
Use $\text{period} = \frac{2\pi}{b}$
 formula to solve for f : $0.0025 \text{ s} = \frac{2\pi}{2\pi f} \Rightarrow \frac{1}{f} = 0.0025 \text{ s} \Rightarrow f = \frac{1}{0.0025 \text{ s}}$
 $f = 400 \text{ Hz}$

ANSWER: **D**



For c_{sine} determine x-coords of max / mins...

20 rotations in 60 sec... (1 min)
 1 rotation in $\frac{60}{20}$
 = 3 sec (this is the period)

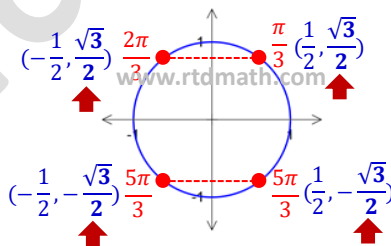


ANSWER: **A**

13. Solve by factoring: $2\cos^2\theta + 3\cos\theta - 2 = 0$
 $(2\cos\theta - 1)(\cos\theta + 2) = 0$
 Two numbers must mult. to -2
 $(2\cos\theta - 1)(\cos\theta + 2) = 0$
 Now set each factor to zero ...
 $2\cos\theta - 1 = 0$ $\cos\theta + 2 = 0$
 ANSWER: **A** $\cos\theta = \frac{1}{2}$ or $\cos\theta = -2$

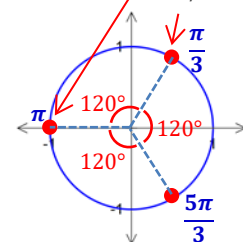
14. First isolate trig term: $3\csc^2\theta - 4 = 0$
 $3\csc^2\theta = 4$
 $\csc^2\theta = \frac{4}{3}$
 Sq. root both sides $\csc\theta = \pm\sqrt{\frac{4}{3}}$
 Since $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$ $\csc\theta = \pm\frac{2}{\sqrt{3}}$
 Since $\csc\theta$ is reciprocal of $\sin\theta$ $\sin\theta = \pm\frac{\sqrt{3}}{2}$

Find where on unit circle the y-coord is $\pm\frac{\sqrt{3}}{2}$



ANSWER: **B**

15. Factor $\sec^2x - \sec x - 2 = 0$
 $(\sec x + 1)(\sec x - 2) = 0$
 Set each factor to zero:
 $\sec x = -1$ or $\sec x = 2$
 Since $\sec x$ is reciprocal of:
 $\cos x = -1$ or $\cos x = \frac{1}{2}$
 Find where on unit circle the x-coord is -1 or 1/2



Note that all three solutions are 120° , or $\frac{2\pi}{3}$, apart

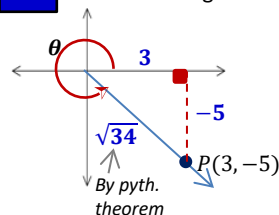
$\theta = \frac{\pi}{3} + \frac{2\pi}{3}n; n \in \mathbb{I}$
 first solution \uparrow diff between sols

ANSWER: **C**

16. First isolate and use log laws on LS
 $\log_2(\tan x) + \log_2(\cos x) = -1$
 $\log_2(\tan x * \cos x) = -1$
 Use trig identities to simplify
 $\log_2\left(\frac{\sin x}{\cos x} * \cos x\right) = -1$
 $\log_2(\sin x) = -1$
 Convert to log form
 $2^{-1} = \sin x \Rightarrow \sin x = \frac{1}{2}$
 Find where on unit circle the y-coord is $\frac{1}{2}$
 $x = \frac{\pi}{6}$ or $\frac{5\pi}{6}$
 ANSWER: **C**

NR#6 First re-write in degrees: $\cos(105^\circ)$
 Next find any two standard unit circle angles that add (or subtract) to 105° Such as: $\cos(45^\circ + 60^\circ)$
 Note: There are many options, another is $\cos(135^\circ - 30^\circ)$
 $\cos(\alpha + \beta) = \cos\alpha \cos\beta - \sin\alpha \sin\beta$
 $= \cos 45^\circ \cos 60^\circ - \sin 45^\circ \sin 60^\circ$
 Finally refer to unit circle and simplify
 $= \left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right) - \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right)$
 $= \frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{4} \Rightarrow \frac{\sqrt{2} - \sqrt{6}}{4}$
 ANSWER: **264**

17. Sketch the angle θ and draw a triangle to determine trig ratios:



Now, for $\sin(\pi - \theta) \dots$

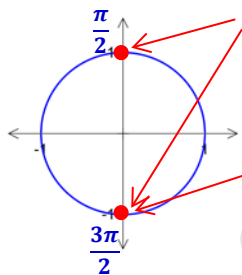
$= \sin\pi \cos\theta - \cos\pi \sin\theta$
 $= (0)\left(\frac{3}{\sqrt{34}}\right) - (-1)\left(\frac{-5}{\sqrt{34}}\right) \Rightarrow = -\frac{5}{\sqrt{34}}$
 From unit circle \uparrow From diagram \uparrow
 $\cos = \frac{\text{adj}}{\text{hyp}}$
 ANSWER: **D**

18. $\frac{\tan x}{1 + \sin x}$ Simplifies to ... $\frac{\sin x}{1 + \sin x}$

So, NPVs ...

$\cos x \neq 0$

$1 + \sin x \neq 0$



$\cos x \neq 0$
Where on the unit circle is the x-coord 0?

$1 + \sin x \neq 0$

$\sin x \neq -1$

Where on the unit circle is the y-coord equal to -1? (already covered above!)

So, NPV at the top / bottom of the unit circle

$x \neq \frac{\pi}{2}$ then every π

Which we write as:

$x \neq \frac{\pi}{2} + n\pi$ where $n \in \mathbb{I}$

ANSWER: C

NR#7 Simplify: $\tan x + \frac{\cos x}{1 + \sin x}$

$$\begin{aligned} &= \frac{\sin x}{\cos x} + \frac{\cos x}{1 + \sin x} \\ &= \frac{\sin x(1 + \sin x)}{\cos x(1 + \sin x)} + \frac{\cos x(\cos x)}{1 + \sin x(\cos x)} \\ &\quad \text{Pyth. identity, this is "1"} \\ &= \frac{\sin x + \sin^2 x + \cos^2 x}{\cos x(1 + \sin x)} \\ &= \frac{\sin x + 1}{\cos x(1 + \sin x)} \\ &= \frac{1}{\cos x} \\ &= \sec x \quad \text{Code: 4} \end{aligned}$$

Simplify: $\frac{\csc x}{\sin x} + \frac{\cot x}{\tan x}$

$$\begin{aligned} &= \frac{1}{\sin x} + \frac{\cos x}{\sin x \cos x} \\ &= \frac{1}{\sin x} * \frac{1}{\sin x} + \frac{\cos x}{\sin x} * \frac{\cos x}{\sin x} \\ &= \frac{1}{\sin^2 x} + \frac{\cos^2 x}{\sin^2 x} \\ &= \frac{1 + \cos^2 x}{\sin^2 x} \\ &= \frac{\sin^2 x}{\sin^2 x} \Rightarrow = 1 \quad \text{Code: 1} \end{aligned}$$

ANSWER: 41

19. Start with appropriate addition / subtraction formula:

$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$

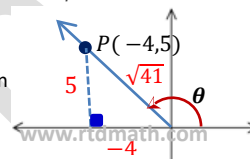
$= \sin \frac{\pi}{3} \cos \theta + \cos \frac{\pi}{3} \sin \theta$

Draw θ using info given

Use unit circle for this and $\cos \frac{\pi}{3}$

$$\begin{aligned} &= \left(\frac{\sqrt{3}}{2}\right) \left(\frac{-4}{\sqrt{41}}\right) + \left(\frac{1}{2}\right) \left(\frac{5}{\sqrt{41}}\right) \\ &= \frac{-4\sqrt{3}}{2\sqrt{41}} + \frac{5}{2\sqrt{41}} \\ &= \frac{-4\sqrt{3} + 5}{2\sqrt{41}} \end{aligned}$$

ANSWER: B



To sketch θ , use: $\cot \theta = -\frac{4}{5}$ adj opp

In **Quad II** since adj side is neg and $\sin \theta > 0$

Use pyth. theorem for hyp., $(-4)^2 + 5^2 = \text{hyp}^2$

so... $\cos \theta = \frac{-4}{\sqrt{41}} \quad \sin \theta = \frac{5}{\sqrt{41}}$

20. **Step 1**
The mistake is here!

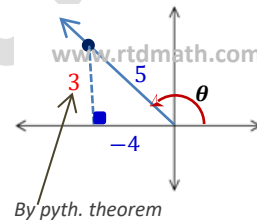
Correct steps:

$$\begin{aligned} &\frac{1 + \sin A - (\cos^2 A - \sin^2 A)}{\cos A + 2 \sin A \cos A} \\ &= \frac{1 - \cos^2 A + \sin A + \sin^2 A}{\cos A + 2 \sin A \cos A} \\ &= \frac{\sin^2 A + \sin A + \sin^2 A}{\cos A + 2 \sin A \cos A} \\ &= \frac{2 \sin^2 A + \sin A}{\cos A + 2 \sin A \cos A} \\ &= \frac{\sin A(2 \sin A + 1)}{\cos A(1 + 2 \sin A)} \\ &= \tan A \end{aligned}$$

ANSWER: A

NR#8

Sketch:



By pyth. theorem

So, $\tan \theta = -\frac{3}{4}$

Formula sheet: $\tan(2\alpha) = \frac{2 \tan \alpha}{1 - \tan^2 \alpha}$

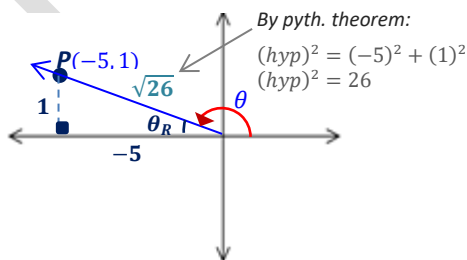
$$\begin{aligned} &= \frac{2 \left(-\frac{3}{4}\right)}{1 - \left(-\frac{3}{4}\right)^2} \\ &= \frac{-\frac{6}{4}}{1 - \frac{9}{16}} \\ &= \frac{-\frac{3}{2}}{\frac{7}{16}} \end{aligned}$$

ANSWER: 247

$= -\frac{3}{2} * \frac{16}{7} \Rightarrow = -\frac{24}{7}$

Written #1

First bullet



By pyth. theorem:

$(\text{hyp})^2 = (-5)^2 + (1)^2$
 $(\text{hyp})^2 = 26$

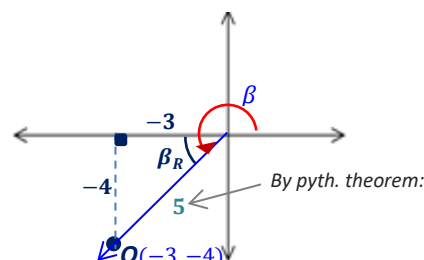
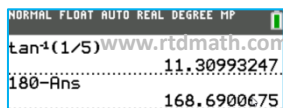
To find θ , first find reference angle θ_R (inside the triangle)

$\theta_R = \tan^{-1}(1/5)$ or $\sin^{-1}(1/\sqrt{26})$ or $\cos^{-1}(5/\sqrt{26})$

so ...

$\theta \approx 180^\circ - 11^\circ$

$\theta \approx 169^\circ$



By pyth. theorem:

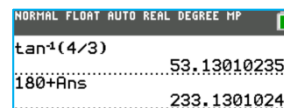
To find β , first find reference angle β_R (inside the triangle)

$\beta_R = \tan^{-1}(4/3)$ or $\sin^{-1}(4/5)$ or $\cos^{-1}(3/5)$

so ...

$\theta \approx 180^\circ + 53^\circ$

$\theta \approx 233^\circ$



WR #1

Second bullet

Refer to your formula sheet:

$$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$$

We'll need all this: (leave exact)

$$\sin \theta = \frac{1}{\sqrt{26}} \leftarrow \text{opp} \leftarrow \text{hyp}$$

$$\cos \theta = \frac{-5}{\sqrt{26}} \leftarrow \text{adj} \leftarrow \text{hyp}$$

similarly...

$$\sin \beta = \frac{-4}{5} \quad \cos \beta = \frac{-3}{5}$$

So...

$$\sin(\theta + \beta) = \left(\frac{1}{\sqrt{26}}\right)\left(\frac{-3}{5}\right) + \left(\frac{-5}{\sqrt{26}}\right)\left(\frac{-4}{5}\right) \Rightarrow = \frac{-3}{5\sqrt{26}} + \frac{20}{5\sqrt{26}}$$

$$\Rightarrow = \frac{17}{5\sqrt{26}}$$

Note: There was a mistake in some answer keys!

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Written #2

Use $\sin^2 \theta + \cos^2 \theta = 1$, which re-arranges to $\sin^2 \theta = 1 - \cos^2 \theta$, to re-write equation in terms of $\cos \theta$ only

First bullet

$$2(1 - \cos^2 \theta) - \cos x - 1 = 0$$

$$2 - 2\cos^2 \theta - \cos x - 1 = 0$$

$$0 = 2\cos^2 \theta + \cos x - 1 \Rightarrow 2\cos^2 \theta + \cos x - 1 = 0$$

2nd bullet

$$(2\cos \theta - 1)(\cos \theta + 1) = 0 \text{ Factor}$$

Find two #'s that mult to -1, and expression will expand back out to $2\cos^2 \theta + \cos x - 1$

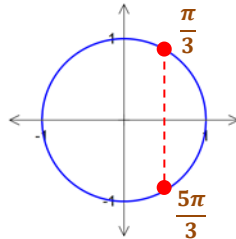
$$(2\cos \theta - 1)(\cos \theta + 1) = 0$$

Set each factor to zero

$$2\cos \theta - 1 = 0 \text{ or } \cos \theta + 1 = 0$$

$$\cos \theta = 1/2$$

Where on the unit circle is the x-coord 1/2?

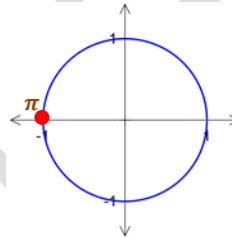


PRIMARY SOLUTIONS:

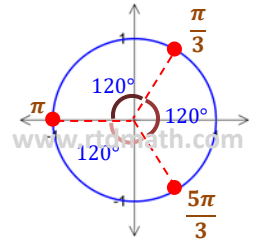
$$\theta = \frac{\pi}{3}, \pi, \frac{5\pi}{3}$$

$$\cos \theta = -1$$

Where on the unit circle is the x-coord -1?



For GENERAL SOLUTION, consider all three primary solutions together



They're all $\frac{2\pi}{3}$ (that is, 120°) apart

GENERAL SOLUTION:

$$\theta = \frac{\pi}{3} + \frac{2\pi}{3}n \quad n \in \mathbb{I}$$

Written #3

Proceed on left side:

First bullet

$$= \frac{1}{\sin x} * \frac{\cos x}{\cos x + \sin x}$$

Write each expression in terms of sin and cos

$$= \frac{\frac{\cos x}{\sin x}}{\frac{\cos x}{\cos x * \sin x} + \frac{\cos x}{\sin x * \cos x}}$$

Re-write bottom expressions with a common denom.

$$= \frac{\frac{\cos x}{\sin x}}{\frac{\cos x}{\sin^2 x + \cos^2 x} \cdot \frac{\cos x \sin x}{\cos x \sin x}}$$

Pyth. identity, this is "1"!

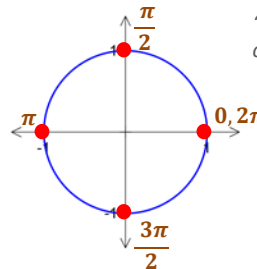
$$= \frac{\cos x}{\sin x} * \frac{\cos x \sin x}{1} \text{ Invert and multiply } \Rightarrow = \cos^2 x$$

2nd bullet

$$= \frac{1}{\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}}$$

Go back to first step, restriction at any expression in the denominator. (Can't divide by 0)

$$\sin x \neq 0 \quad \cos x \neq 0$$

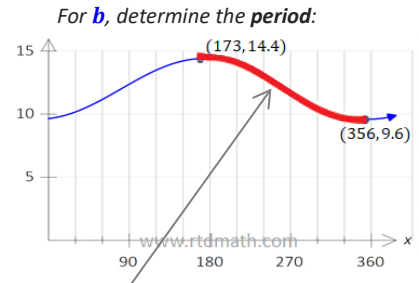
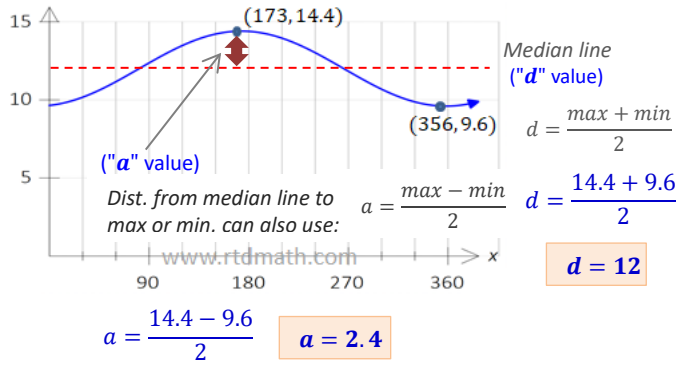


"where on the unit circle is the x or y coord equal to 0?"

$$x \neq \frac{\pi}{2}n \quad n \in \mathbb{I}$$

Written #4

First bullet



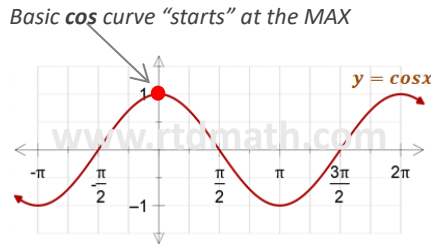
This shows half of the **period** (dist from max to min)

Half of Period = 183 days (356 - 173)

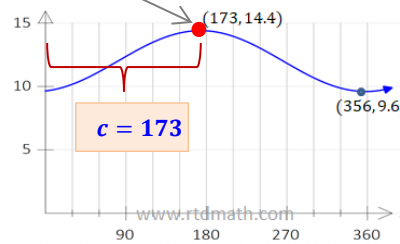
Period = 366 days Actually, this was given!

$$b = \frac{2\pi}{366} \Rightarrow b = \frac{\pi}{183}$$

Finally for **c_{cos}**, determine the horiz. dist of the max from the y-axis



Here the max has **shifted**, 173 to the right



$$H = 2.4 \cos\left[\frac{\pi}{183}(x - 173)\right] + 12$$

2nd bullet

Graph $y_1 = 5.1 \sin(0.524(x - 2.75)) + 23.9$

And find the **INTERSECTS**

$y_2 = 26$

WINDOW
 Xmin=0
 Xmax=12
 Xscl=1
 Ymin=0
 Ymax=30
 Yscl=1
 Xres=1

For x-max, graph over 1 period

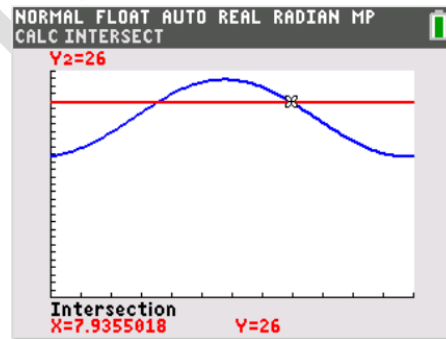
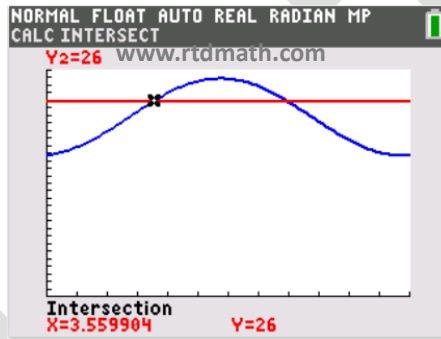
$\text{per} = \frac{2\pi}{0.524}$

≈ 12

For y-max, use **MAX = a + d**

$= 5.1 + 23.9$

$= 29$



So, total # months above 26°C can be found by subtracting...

$= 7.94 - 3.56$

4.4 months

Third bullet

a would be **higher**, as the range of Calgary temperatures (between min and max) would be greater

d would be **lower**, as the median temperature for Calgary (represented by **d**) would be lower

Also.... (not needed in your answer)

b would be **unchanged**, as the period for each city would be the same (12 months). Similarly, **c** would be essentially unchanged, as the number of months after which the min / max temperature occurs would be approximately the same as both cities are in the northern hemisphere.

